5.10. Translation Complications: Universals (Again)

1. Universals with Multiple Predicates. Multi-predicate existential sentences made for easy formal translation, since they employed conjunctions, where neither order nor grouping of parts is relevant to truth and validity. More care is called for with multi-predicate universals, however; for these involve conditionals, where order and grouping make a difference. We noted in Chapter Four that "$(P \rightarrow Q)$" does not mean the same as "$(Q \rightarrow P)$," and that "$((P \rightarrow Q) \rightarrow R)$" likewise differs in meaning from "$(P \rightarrow (Q \rightarrow R))$".

But as a good rule of thumb, grouping of predicates here follows the divide between the grammatical **subject** and **predicate** of the English sentence. Taken in the grammatical sense, the subject states the actor or topic of the sentence, while the predicate states the action performed or feature possessed. So, e.g., in the sentence "All good athletes like exercise," the grammatical subject is "good athletes" and the grammatical predicate is "like(s) exercise".

In universal sentences the subject will form the **antecedent** of the conditional, while the predicate forms the **consequent**. So in the universal sentence "All black cats are lucky" the subject is "black cats," while the predicate is "lucky". The divide between these two parts of the sentence is where the arrow appears.

All **black cats** are **lucky**

The stacked-up predicates "black cats" form a conjunction, serving as the antecedent of the conditional; while "is lucky" appears in the consequent.

For all x: if *x is black and x is a cat,* *then x is lucky*

G: is black **I**: is lucky
H: is a cat

$$\forall x\, ((Gx \wedge Hx) \rightarrow Ix)$$

In the next example the subject of the sentence is just "cats," while the grammatical predicate features the conjoined phrases "eat meat" and "drink milk". So the consequent is likewise a conjunction.

All **cats** **eat meat** and **drink milk**.

G: is a cat **I**: drinks milk
H: eats meat

$$\forall x\,(Gx \rightarrow (Hx \wedge Ix))$$

But we get a curious result when a conjunction appears in the *subject* of an English universal sentence – as in this example.

All **children and adults** will enjoy the movie.

Certainly it is a mistake to treat this sentence as first restricting the discussion to a select group – the things which are *both children and adults* – and saying of those things that they will enjoy the movie. The English sentence isn't naturally read as making a claim about such an impossible sort of thing (the child-adults), but rather as a **conjunction** of **two universals**.

All children will enjoy the movie, **and** all adults will enjoy the movie.

The original English sentence can indeed be accurately translated by treating it as this conjunction.

G: is a child **I**: will like the movie
H: is an adult

$$(\underline{\forall x\,(Gx \rightarrow Ix)} \;\wedge\; \underline{\forall x\,(Hx \rightarrow Ix)}\,)$$

And since the conjoined sentences are both universals, "x" applies consistently to every object. That allows us to conjoin "(Gx → Ix)" and "(Hx → Ix)" within the scope of a *single* universal quantifier without change of meaning.

The following two sentences are indeed equivalent.

$$\mathbf{(\forall x\ (\underline{Gx \rightarrow Ix})\ \wedge\ \forall x\ (\underline{Hx \rightarrow Ix})\ )}$$
$$\mathbf{\forall x\ (\ (\underline{Gx \rightarrow Ix}) \wedge (\underline{Hx \rightarrow Ix})\ )}$$

But familiarity with conditionals offers a shorter equivalent. Recall that when two conditionals have the same consequent, the conjunction of those two sentences is equivalent to a single conditional: with the same consequent, and the **disjunction** of the two antecedents. “$((P \rightarrow R) \wedge (Q \rightarrow R))$,” for instance, is equivalent to “$((P \vee Q) \rightarrow R)$”.

P	Q	R	$(P \rightarrow R)$	$(Q \rightarrow R)$	$\mathbf{((P \rightarrow R) \wedge (Q \rightarrow R))}$	$(P \vee Q)$	$\mathbf{((P \vee Q) \rightarrow R)}$
1	1	1	1	1	**1**	1	**1**
1	1	0	0	0	**0**	1	**0**
1	0	1	1	1	**1**	1	**1**
1	0	0	0	1	**0**	1	**0**
0	1	1	1	1	**1**	1	**1**
0	1	0	1	0	**0**	0	**0**
0	0	1	1	1	**1**	1	**1**
0	0	0	1	1	**1**	0	**1**

The two sentences are *intuitively* equivalent as well – as English examples show.

> If we have ham we can make a sandwich; and if we have turkey we can make a sandwich.

> If we have either ham or turkey, we can make a sandwich.

That equivalence maps onto universal sentences – unsurprisingly, since they use conditionals. So the following two translations will be logically equivalent.

G: is a child **I**: will like the movie
H: is an adult

$$\forall x\ (\ (Gx \rightarrow Ix) \wedge (Hx \rightarrow Ix)\ )$$
$$\forall x\ ((Gx \vee Hx) \rightarrow Ix)$$

This equivalence is most obvious in English when using a tacit (unspoken) universal quantifier. For the following sentences *do* seem to say the same thing.

Children and adults will enjoy the movie.
If you are either a child or an adult, you will enjoy the movie.

(In the second sentence the pronoun "you" acts as an English variable, on a par with "it".)

A multi-predicate variant on "only" sentences comes in "Gs are the only Hs that I". The following sentences are translated into the same formal sentence.

$$\left.\begin{array}{l}\text{Bats are the only mammals that have wings.}\\ \text{Among mammals, only bat have wings.}\\ \text{Only bats are mammals having wings.}\\ \text{All mammals having wings are bats.}\end{array}\right\}\ \forall x\ ((Gx \wedge Ix) \rightarrow Hx)$$

G: is a mammal **I**: has wings
H: is a bat

Note that the phrasing "among G" serves as an antecedent of a conditional – so that "Among G, all H are I" is translated as "$\forall x\ (Gx \rightarrow (Hx \rightarrow Ix))$". So "Among G, only H are I" will be translated as "$\forall x\ (Gx \rightarrow (Ix \rightarrow Hx))$" (because of the familiar order-switching effect of "only").

But in fact “$\forall x\ (Gx \rightarrow (Ix \rightarrow Hx))$” is equivalent to our earlier sentence “$\forall x\ (Gx \rightarrow (Ix \rightarrow Hx))$” – again, because of an equivalence from a previous chapter. Recall that “$(P \rightarrow (Q \rightarrow R))$” is logically equivalent to “$((P \wedge Q) \rightarrow R)$”.

P	Q	R	$(Q \rightarrow R)$	$\mathbf{(P \rightarrow (Q \rightarrow R))}$	$(P \wedge Q)$	$\mathbf{((P \wedge Q) \rightarrow R)}$
1	1	1	1	**1**	1	**1**
1	1	0	0	**0**	1	**0**
1	0	1	1	**1**	0	**1**
1	0	0	1	**1**	0	**1**
0	1	1	1	**1**	0	**1**
0	1	0	0	**1**	0	**1**
0	0	1	1	**1**	0	**1**
0	0	0	1	**1**	0	**1**

So “Among G, only I are H,” and all its translation variants, can be translated either way.

$$\left.\begin{array}{l}\text{Bats are the only mammals that have wings.}\\ \text{Among mammals, only bat have wings.}\\ \text{Only bats are mammals having wings.}\\ \text{All mammals having wings are bats.}\end{array}\right\}\ \begin{array}{l}\mathbf{\forall x\ ((Gx \wedge Ix) \rightarrow Hx)}\\ \mathbf{\forall x\ (Gx \rightarrow (Ix \rightarrow Hx))}\end{array}$$

G: is a mammal **I**: has wings
H: is a bat

2. Universals, Existentials, and Negation. Mixing negations into quantified sentences largely repeats the variations rehearsed above – though with the occasional curveball thrown in.

In an existential sentence with complex clusters of predicate phrases – featuring, for example, negations or disjunctions of predicate phrases – the divide between subject and predicate positions is a good clue.

G: is an animal	**J**: is a student
H: is a mammal	**K**: passed the exam
I: is a lizard	**L**: studied

Some animals are neither mammals nor lizards.

$$\exists x\,(Gx \wedge \sim(Hx \vee Ix))$$

Some students passed the exam without studying.

$$\exists x\,(Jx \wedge (Kx \wedge \sim Lx))$$

We appeal to this divide as well in translating universal negative sentences.

No lizards are either mammals or birds.

Since its formal counterpart employs a conditional with negated consequent, the above sentence is translated like so.

G: is a lizard	**I**: is a bird
H: is a mammal	

$$\forall x\;(Gx \rightarrow \sim(Hx \vee Ix))$$

Cases of tacit quantification provide support here; for these two sentences are (rightly) translated the same way.

No lizards are **either mammals or birds**.
Lizards are **neither mammals nor birds**.

The following sentence poses a trickier translation, however.

Neither mammals nor birds are lizards.

Treating this (correctly) as a case of tacit universal quantification, we may be tempted to translate like so.

☠ Proper Translation?? ☠

G: is a mammal **I**: is a lizard
H: is a bird

Neither mammals nor birds ⋮ **are lizards**.

For all x, if x is neither a mammal nor a bird, ⋮ then x is a lizard.

$$\forall x\ (\sim(Gx \vee Hx) \rightarrow Ix)$$

But that formal sentence makes a claim *far stronger* than the English one – saying that *anything which is neither mammal nor bird is a lizard*. Rocks, for example, are neither mammal nor bird; so the formal sentence counts them as lizards. But the English sentence "Neither mammals nor birds are lizards" certainly doesn't count rocks as lizards. This formal translation is not capturing the meaning of the English original.

Here we need to recall once again that with a universal negative sentence, formal translation kicks the negation into the associated consequent. Since "neither… nor" is the negation of "either… or," when that negation is kicked into the consequent we are left with "either… or" in the antecedent.

Neither mammals nor birds ⋮ **are lizards**.

$$\forall x\ ((Gx \vee Hx) \rightarrow \sim Ix)$$

As a bit of confirmation, note that the following two sentences means the same thing – and that the second clearly takes the formal translation we're recommending.

G: is a mammal **I**: is a lizard
H: is a bird

Neither mammals nor birds are lizards.

$$\forall x\ ((Gx \lor Hx) \to {\sim}Ix)$$

Anything which is either a mammal or a bird is not a lizard.

$$\forall x\ ((Gx \lor Hx) \to {\sim}Ix)$$

And here recurs a point noted earlier: a conditional with a disjunction for its antecedent is equivalent to a conjunction of two conditionals.

"$((P \lor Q) \to R)$" is equivalent to "$((P \to R) \land (Q \to R))$".

For that reason, these two universal sentences are equivalent.

$$\forall x\ ((Gx \lor Hx) \to {\sim}Ix)$$
$$(\forall x\ (Gx \to {\sim}Ix) \land \forall x\ (Hx \to {\sim}Ix))$$

And the English counterparts to these sentences (each with a tacit universal quantifier) *do* intuitively seem to make the same claim.

Neither mammals nor birds are lizards.
Mammals are not lizards, and birds are not lizards.